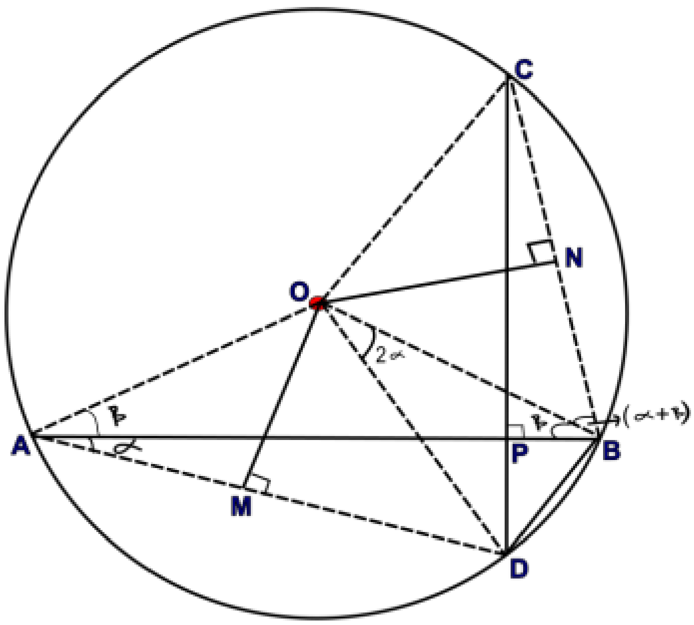
Hara Gopal



Supporting Result to prove the problem

AB & CD are ⊥ chords, then $AP^2 + BP^2 + CP^2 + DP^2 = 4r^2$

Proof:

Let $\angle BOD = 2\alpha,$ then

$\angle BCP = \angle BAD = \alpha$ [Angle at center is twice the angle of any point of the circle]

$\Rightarrow \angle CBA = (90 - \alpha)$

Let $\angle OAB = \beta = \angle OBA$ [$\because OA = OB$]

$\Rightarrow \angle OBC = 90 - (\alpha + \beta)$

Now draw OM ⊥ AD & ON ⊥ BC

In ΔOAM & ΔBON

$\angle O = \angle B = 90 - (\alpha + \beta)$

$\angle M = \angle N = 90°$

$\therefore \Delta OAM \sim \Delta BON$

$\frac{OA}{BO} = \frac{AM}{ON} = \frac{OM}{BN}$

Since, OA =OB= radius

AM = ON & OM = BN

Now in ΔOAM

$OA^2 = AM^2 + OM^2$

$\alpha^2 = AM^2 + BN^2$

$\alpha^2 = \left(\frac{AD}{2}\right)^2 + \left(\frac{BC}{2}\right)^2$

$4\alpha^2 = AD^2 + BC^2$

$4\alpha^2 = AP^2 + DP^2 + BP^2 + CP^2$ ------------------ Hence Proved.

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From the result proved in the given figure

Let the point of intersections of the chords be 'N', then

$AN^2 + BN^2 + CN^2 + EN^2 = 4r^2$

$= (2r)^2$

$= DF^2$

$= CF^2 + CD^2$ [As angle in a semi circle is 90°]

$(AM + MN)^2 + (BM - MN)^2 + CN^2 + EN^2 = CF^2 + CD^2$

$AM^2 + MN^2 + +2AM.MN + AM^2 + MN^2 - 2AM.MN + CN^2 + EN^2 = CF^2 + CD^2$ $[\because AM = MB]$

$2AM^2 + 2MN^2 + CN^2 + EN^2 = CF^2 + CD^2$

$2AM^2 + ME^2 + CM^2 = CF^2 + CD^2$

$2AM^2 + ME^2 + CM^2 = CF^2 + (CM + MD)^2$

$2AM^2 + ME^2 + CM^2 = CF^2 + CM^2 + MD^2 + 2CM.MD$

$2AM^2 + ME^2 - 2CM.MD = CF^2 + MD^2$ -------------------(1)

Now as points A, D, B.C are cyclic

$\Delta AMD \sim \Delta CMB$

$\Rightarrow \frac{AM}{CM} = \frac{MD}{MB}$

AM x MB = CM x MD

AM x AM = CM x MD [since 'M' is midpoint of AB]

$\Rightarrow AM^2 = CM \; X \; MD$

Substituting the above in (1)

$2AM^2 + ME^2 - 2CM \times MD = CF^2 + MD^2$

$2AM^2 + ME^2 - 2AM^2 = CF^2 + MD^2$

$\Rightarrow ME^2 = CF^2 + MD^2$

which means CF, MD, ME forms a right triangle. -------- Hence Proved.

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